EUROPHYSICS LETTERS 1 March 1997

Europhys. Lett., **37** (7), pp. 459-463 (1997)

Instabilities of ion confinement in a penning trap

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(received 14 November 1996; accepted in final form 24 January 1997)

PACS. 52.25Wz – Nonneutral plasmas.
PACS. 52.20Dq – Particle orbits.
PACS. 05.45+b – Theory and models of chaotic systems.

Abstract. – We have observed rapid ion loss from a Penning trap at certain operating conditions. Here the ratio of the cyclotron or axial oscillation frequency is an integer multiple of the "metastable" magnetron frequency. Higher-order contributions to the ideal quadrupole trapping potential causes energy transfer into the magnetron motion, whose orbit increases. This effect is analogous to the stop bands observed in accelerators and storage rings.

Introduction. – Penning traps confine particles of charge q and mass m to a limited volume in space by an electric potential of the form

$$\phi = \frac{U}{2r_0^2}(x^2 + y^2 - 2z^2) \tag{1}$$

and a superimposed magnetic field B directed along the z-axis. The potential is created by a static voltage U applied to three hyperbolic-shape electrodes, one ring of radius r_0 and two endcaps located at $r_0/\sqrt{2}$ above and below the trap center. The special shape of (1) is chosen to have (for proper sign of U) a harmonic axial binding force, while ion loss in the radial plane is prevented by the magnetic field. The classical equation of motion in such a potential can be solved analytically [1]. Along the z-axis we have a harmonic oscillation of frequency

$$\omega_z = \left[\frac{2qU}{mr_0^2}\right]^{1/2}. \tag{2}$$

In the radial plane the motion is a superposition of the cyclotron motion, slightly perturbed by the presence of the electric trapping field, of frequency

$$\omega_{\mathrm{c}}' = \frac{\omega_{\mathrm{c}}}{2} + \left[\frac{\omega_{\mathrm{c}}^2}{4} - \frac{\omega_z^2}{2}\right]^{1/2} \tag{3}$$

and a so-called magnetron motion of frequency

$$\omega_{\mathrm{m}} = \frac{\omega_{\mathrm{c}}}{2} - \left[\frac{\omega_{\mathrm{c}}^2}{4} - \frac{\omega_z^2}{2}\right]^{1/2}; \tag{4}$$

$\omega_{\mathrm{c}} = (q/m)B$ is the free-ion cyclotron frequency.

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Stable confinement is achieved for positive values of the bracket in eqs. (3), (4), when the voltage U and the superimposed magnetic field B are related as

$$U < \frac{qr_0^2}{4m} B \,. \tag{5}$$

Equations (2)-(4) hold for a single particle in a perfect trap with hyperbolic-shape electrodes. A cloud of ions modifies the potential by space charge. Moreover, non-perfect hyperbolic shape of the electrodes, misaligments or modification of the electrodes by holes lead to deviation from the ideal quadrupole potential (1). If we assume rotational symmetry we can expand the real potential ϕ into a power series of spherical harmonics [1]:

$$\phi = \frac{\phi_0}{2} \sum_{k=0}^{\infty} C_k \left(\frac{\rho}{r_0}\right)^k P^k(\cos\vartheta) \,. \tag{6}$$

Here we have used spherical coordinates ρ, ϑ. P^k is the spherical harmonic of order k and C_k denotes the strength of the multipole component of order k. An important effect of the perturbing potentials are shifts of the eigenfrequencies (2)-(4) with respect to the ideal quadrupole trap. These shifts depend on the size of the coefficients C_k and on the amplitude of the ion oscillations. They have been calculated by different authors [1]-[4] and are of great concern in precision mass spectroscopy using Penning ion traps.

A different result of higher-order components in the trap potential is the coupling of the three different oscillation modes, which in a perfect Penning trap would be completely uncoupled. As a consequence linear combinations of the eigenfrequencies, given by eqs. (2)-(4), can be observed experimentally [5]. Also at certain operating points of the trap energy exchange can take place between the different oscillations. This happens when one of the eigenfrequencies is a harmonic of another. This is of particular importance if one of the involved oscillations is the magnetron motion. It describes the motion of the center of the cyclotron orbit around the trap center in the x-y plane. Its energy is almost exclusively given by the potential energy in the trap's electric field. It decreases with increasing radius of the magnetron orbit. In the Penning trap there is no restoring force in the radial direction, rather there is a balance between the electrostatic force directed away from the axis and a magnetic $(v \times B)$ force directed towards the axis. If the magnetron frequency is a subharmonic of the axial or cyclotron frequency, energy is coupled into the magnetron motion, its radius increases and eventually the particle gets lost from the trap.

Indications of such instabilities have been observed in an experiment by Yu *et al.* [6] who noted jumps in the trapping efficiency of a Penning trap at certain values of the applied magnetic field. More recently a direct observation of an instability has been reported by Schweikhard *et al.* [7] who measured a decrease of the trap's trapping efficiency near the limit of stability. They calculated ratios of $\omega_z/\omega_\mathrm{m} = \omega_\mathrm{c}'/\omega_\mathrm{m} = 2$ for the unstable condition. A related effect has recently been noted in an extended Penning-type trap for positron accumulation [8]. Here an enhanced positron accumulation rate has been observed, when the cyclotron orbiting time was an odd integer of the axial oscillation period.

Experiment and discussion. – In this contribution we report about the observation of several unstable resonances in a laser-spectroscopic experiment on Ba^+ ions in a Penning trap. Our trap had a radius of 13 mm and was placed in a superconducting solenoid of 2.9 T strength. The ions are irradiated by a broad band (1 GHz) pulsed dye laser at their $6S_{1/2}$-$6P_{1/2}$ resonance wavelength (493 nm). The laser-induced fluorescence at the same wavelength was observed perpendicular to the laser beam through a mesh endcap electrode. The excited $6P_{1/2}$ level may also decay into a long-living ($\tau = 48$ s) metastable $5D_{3/2}$ state. Repumping from this

TABLE I. – *Operating voltages $U_{\mathrm{c(exp)}}$ of a Penning trap (r_0 = 13 mm, B = 2.9 T) where rapid ion loss from the trap was observed. They agree with calculated voltages $U_{\mathrm{c(theo)}}$ where the ratio of eigenfrequencies $\omega_z/\omega_{\mathrm{m}}$ or $\omega'_{\mathrm{c}}/\omega_{\mathrm{m}}$ is an integer number.*

$\omega_z/\omega_{\mathrm{m}}$	$U_{\mathrm{c(exp)}}$ (V)	$U_{\mathrm{c(theo)}}$ (V)	$\omega'_{\mathrm{c}}/\omega_{\mathrm{m}}$	$U_{\mathrm{c(exp)}}$ (V)	$U_{\mathrm{c(theo)}}$ (V)
3	146 (2)	147.5	4	158 (1)	158.7
4	98 (1)	97.9	5	129 (1)	137.7
			8	98 (1)	97.9
			9	87 (2)	89.3

state would substantially increase the observed fluorescence intensity. Because of the large Zeeman splitting of 27 GHz and 32 GHz for the $6P_{1/2}$ and $5D_{3/2}$ levels, respectively, three different lasers at 650 nm would be required. These were not available to us and we relied on the small number of ions which decayed back from the $5D_{3/2}$ level into the ground state during the 100 ms time interval between two laser pulses at 493 nm. We stored approximately 10^4–10^5 ions for times of the order of 30 minutes at background pressures below 10^{-9} mbar. When we scanned the trapping voltage continuously from low to high values we observed a drop of the ionic fluorescence to the scatter level at certain critical voltages U_{c}. Figure 1 shows an example. By creating a stored ion cloud at trapping voltages above U_{c} and by scanning to lower voltages, the same phenomena were observed at the same values of U_{c}, however at different fluorescence intensities since the initial number of stored ions was different. For clarity we show in fig. 1 only one scan direction. In table I we have summarized the observed voltages U_{c} for sudden ion loss in our B-field of 2.9 T. When we calculate the eigenfrequencies of the Ba^+ ions at these operating points we find that the ratios of ω_z or ω'_{c} to ω_{m} are integer numbers (see table I). In the example shown in fig. 1 two conditions for instability coincide: $\omega'_{\mathrm{c}}/\omega_{\mathrm{m}} = 8$ and $\omega_z/\omega_{\mathrm{m}} = 4$.

The effect of the perturbing potential on the ion motion becomes evident when we calculate ion trajectories at those operating points where the ratio of eigenfrequencies becomes an integer. To simplify the calculation we considered only an octupole perturbation, characterized by the coefficient C_4 in eq. (6). The equations of motions for the different coordinates then read, if z is the trap's symmetry axis and $B = B_z$:

$$\begin{cases} m\ddot{x} = eB\dot{y} - (Ue/r_0^2)\left[x + \dfrac{C_4}{r_0^2}(3x^2 + 3xy^2 - 12xz^2)\right], \\ m\ddot{y} = -eB\dot{x} - (Ue/r_0^2)\left[y + \dfrac{C_4}{r_0^2}(3y^3 + 3x^2y - 12yz^2)\right], \\ m\ddot{z} = (Ue/r_0^2)\left[-2z - \dfrac{C_4}{r_0^2}(12xz^2 + 12yz^2 - 8z^3)\right]. \end{cases} \tag{7}$$

The coupling of the different coordinates by the perturbation becomes immediately evident.

Equations (7) were solved for a value of $C_4 = 10^{-3}$ (normalization: $C_2 = 1$). If we choose the magnetic field B and the trapping voltage U such that the ions eigenfrequencies ω'_{c} and ω_{m} (see eqs. (3), (4)) relate as $\omega'_{\mathrm{c}}/\omega_{\mathrm{m}} = 9$, the ion radius increases in time until it strikes the trap electrodes (fig. 2, dashed line). For comparison we show the trajectories for a nearby operating point, $\omega'_{\mathrm{c}}/\omega_{\mathrm{m}} = 8.7$ for identical initial conditions in position and velocity and an identical trap anharmonicity. Here the amplitude of the ion motion remains finite in time. A value of $C_4 = 10^{-3}$ is typical for uncompensated traps and may be caused by the truncated electrodes, by holes of 3 mm diameter opposite to each other in our trap's mid plane for laser entrance and exit, and by a grid endcap electrode for fluorescence light transmission.

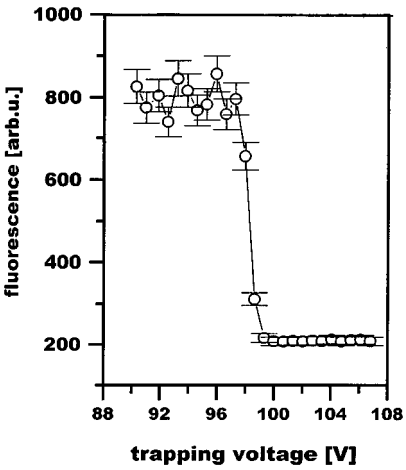

Fig. 1.

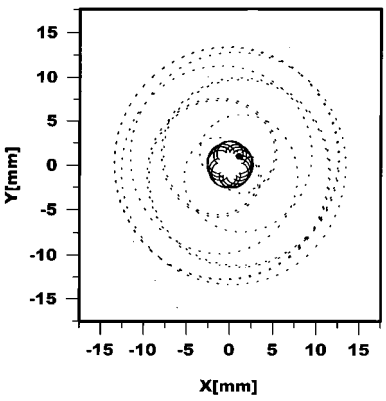

Fig. 2.

Fig. 1. – Observed drop of laser-induced fluorescence intensity from a cloud of Ba^+ ions in a Penning trap when we tune the trap voltage. At the indicated value of $U = 97.9$ V we have an integer ratio of two eigenfrequencies: $\omega_c'/\omega_m = 8$ and simultaneously $\omega_z/\omega_m = 4$.

Fig. 2. – Calculated ion trajectories in the x-y plane of a Penning trap, having an octupole contribution of 0.1% added to the quadrupole trapping potential. For an operating point where $\omega_c'/\omega_m = 9$ we see a rapid increase of the ion orbit (dotted lines) while for a nearby operating point, $\omega_c'/\omega_m = 8.7$ (full line), the ion motional amplitude remains constant. The initial conditions for both trajectories are identical, indicated by a dot.

The actual value of C_4 for our trap may be somewhat different and could in principle be obtained by an accurate measurement of the Ba^+ eigenfrequencies. In addition higher-order perturbations are certainly present in our trap, but their inclusion in the calculation would not change the general behaviour.

Conclusion. – We have observed ion loss from a Penning trap at operating points, when the ratio of the axial or cyclotron frequency of the stored ions to their magnetron frequency is an integer number. Then coupling and energy transfer between the different ion motions takes place caused by anharmonic contributions to the trapping potential. The Penning trap with non-linear forces represents a calculable system that may be used to study the onset of chaotic behaviour in the ion motion as indicated in ref. [8]. The observed energy transfer between the different degrees of freedom in a non-linear Penning trap has its analog in the stop bands of particle storage rings and accelerators where, by periodic perturbations, the ions are eventually lost from the ring [9].

In Penning traps these instabilities can easily be avoided by choosing the proper operating voltages, they can, however, be used to eliminate unwanted ions from the trap. Since the resonance frequencies (2)-(4) are mass dependent, one may choose a trapping voltage, where an ion under investigation remains confined, while a different mass ion could be made unstable. This could increase the maximum number of ions of the preferred species since the total trap capacity is limited by space charge.

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Our experiments are supported by the Deutsche Forschungsgemeinschaft.

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